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Considerations on Spark- Gap Channel Radius and Electrical Conductivity

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K E Y W O R D S

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A B S T R A C T

A simple phenomenological model is established to determine the temporal evolution of spark gap channel radius and electrical conductivity during the resistive phase period. The present determination is based on the Braginskii's equation for the channel radius which includes the electrical conductivity of the discharge channel as a constant quantity. In the present model, however, the electrical conductivity is regarded as a time varyingquantity. Basing on this, a mathematical formulation for the channel radius as a function of time was derived, and this has made possible the derivation of an explicit expression for the conductivity as a function of time as well. Taking the temporal average of the electrical conductivity offers an alternative mathematical formulation for the instantaneous radius based on a steady conductivity value that can be determined according to some experimental parameters. It has been verified that both of the channel radius formulations mentioned above lead to similar results for the temporal evolution. The obtained results of the channel radius were used to determine the instantaneous inductance of the spark channel.

The present model was used to examine the role of gas pressure and gap width on the temporal evolutions of the channel radius, conductivity, and inductance in nanosecond spark gaps.

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1. Introduction

The electrical discharge in gases has attained a large scope of experimental and theoretical work due to its significance both in scientific research and application purposes [1,4]. The electrical discharge in gases occurs when a sufficient amount of energy is supplied to a gaseous medium. This is performed by different methods, and the most popular is by applying sufficient electric potential difference across a gaseous medium confined between two electrodes [5]. The electrical discharge is manifested in different modes depending on experimental conditions, these modes range from non-self-sustaining to arc discharges [5, 6]. In this article, the spark discharge will be considered in particular, which is characterized by relatively high currents and low voltages [5]. Spark may be viewed as a rapid transient discharge that collapses before developing into an arc [1].

In the early stages of electrical discharge, the gaseous medium gets ionized gradually provided the existence of initial free electrons [6, 7]. The populations of free electrons and ions in the gas are increased by different mechanisms like the avalanche process in which more free electrons are generated due to collisions of the accelerated electrons with neutral atoms or molecules. Secondary emissions of free electrons may take place at the cathode surface due to bombardment with positive ions, in addition to other mechanisms like photoemission, thermionic- and field emissions [1, 8, 9]. This stage is known as a non- self- sustaining discharge where the availability of the external source of free electrons is necessary to maintain the ionization process. However, when the electrons and ions populations increase to certain limits the discharge converts into a self- sustained type, which proceeds without the requirement of external sources of free electrons, this phenomenon is known as electrical breakdown [1, 5, 6]. During a breakdown, the resistance of the ionized gas or plasma (also known as the discharge channel) decreases with time as a result of the continuous increase in electron and ion populations. This is accompanied by a drop-in voltage across the discharge (between the electrodes) and a raise in the electrical current passing through [10]. The breakdown interval is also known as the resistive phase since the plasma resistance is dynamic (decreases with time). It is during this phase the spark channel heating takes place due to the growing electrical currents [10] where the gap begins to conduct currents efficiently [1].

In spark gaps, which are mostly employed as fast switching devices (FSDs) [4], the resistive phase is considered as the switching period during which the gap transfers from a non-conducting to a conducting state [10]. Accordingly, the resistive phase is a crucial factor in spark gap operation and it is beneficial to consider the discharge parameters during this phase. Maybe, the most important of those parameters is the dynamic resistance of the discharge channel. Several models have been developed concerning electrical discharge in gases [1, 2, 6]. Each of which has its own assumptions, approximations, and principles. This article takes into consideration the spark channel radius in particular due to its role in determining the time-varying inductance of the discharge [11].

Among the many models; developed to determine and simulate phenomena related to sparks (especially the dynamic resistance of the discharge channel); is the Braginskii's model, in which the radial expansion of the spark channel is taken into consideration, where it is assumed that the Joule heating causes the spark channel to expand radially. However, as Braginskii's model assumes; the channel undergoes cooling due to hydrodynamic and radiation effects. Consequently; the channel temperature does not change appreciably, which implies an approximately steady electrical conductivity ($\sim 2 \times 10^4\ S/m$) of the spark channel [11, 12]. Beside its classification as to yield good qualitative agreements with experiments [11, 13, 14], the Braginskii's model rarely fit with experimental data due to the assumption made of a steady electrical conductivity [1, 11].

In this work, the vision of the Braginskii for the expanding channel is adopted. The channel radius is expressed mathematically as a function of time with the electrical conductivity considered as a time-dependent quantity. This has made possible the formulation of electrical conductivity as a function of time as well. In this article, temporal evolutions of both the spark channel radius and electrical conductivity are determined throughout the period of the resistive phase exclusively, during which the spark gap turns from an open to a closed switch [1]. By taking the temporal average of the conductivity throughout the resistive phase, a second formulation is obtained from the channel radius as a function of time-based on a steady conductivity (which is the averaged conductivity in this case). This is in accordance with Braginskii's assumption of constant conductivity. However, the difference between those two conductivities is that the averaged conductivity introduced in this work is determined based on some physical parameters which may be changed according to experimental circumstances, rather than assigning a restricted value for the conductivity as an approximation [12].

The aim of this work is to attain a mathematical formulation for the time dependence for both of the electrical conductivity and radius of a spark channel. Based on those derived formulations, a computational model is to be constructed that determines the temporal evolution of the radius, conductivity, and inductance for the spark channel during the electrical breakdown period in a spark gap.

2. FUNDAMENTAL EQUATIONS

In this paragraph, the fundamental equations employed in the present model will be demonstrated in brief. The potential difference between the spark gap electrodes at which breakdown occurs is approximated by the Paschen's law given as [1, 8]:

$$V_b = \frac{BPd}{ln(Pd)+C} \quad (1)$$

Where (P) is the gas pressure, (d) the gap width (or electrodes spacing), and the parameters (B) and (C) are determined experimentally and are given for some gases [1, 15, 16].

The temporal dependence of the spark electrical current is expressed via the following equation [17]:

$$i(t) = \frac{d}{A}\sqrt{\frac{3P}{K_R}}\, t^{5/2} \quad (2)$$

Which was extracted from the Rompe and Weizel's model for the spark discharge resistance determination [1]. Here, $A = 0.23\, Z\tau_R^3$, (Z) is the circuit impedance, (K_R) is a constant [1], and (τ_R) is the resistive phase period determined by the empirical equation of Sorensen and Ristic, given for Nitrogen as [18]:

$$\tau_R = \frac{44\, P^{1/2}}{E_o Z^{1/3}} \quad (3)$$

Where (E_o) is the electric field intensity in (10 kV/cm).
The time dependence of the channel resistance is formulated via an empirical equation, also presented by Sorensen and Ristic in the form [1, 18]:

$$R(t) = 0.23\, Z(\tau_R/t)^3 \quad (4)$$

Or, $R(t) = At^{-3}$ (5)

Where R(t) is the discharge resistance and (A) as defined above.
Taking into consideration the discharge channel expands, the channel conductivity (σ) may be expressed as [1, 11]:

$$\sigma = \frac{d}{R(t)\pi a_{(t)}^2} \quad (6)$$

Where a (t) is the instantaneous channel radius.

It should be mentioned that using the Paschen's law for the breakdown potential is a typical procedure that should not necessarily model real experiments [1, 5]. However, employing this law is useful in conducting parametric studies, where it yields the breakdown potential that corresponds to a given set of gas pressure and gap width that fall within the validity of the law [1]. Another approximation made in the present model is employing the mathematical expression for the electric current given by Eq.(2) since the Rompe and Weizel's model; from which this equation was extracted; does not take into account the expansion of the spark channel [1]. Here it should also be mentioned that the empirical equation of Sorensen and Ristic [Eq. (4)] was employed in the derivation of Eq. (2) [17].

3. ELECTRICAL CONDUCTIVITY AND SPARK CHANNEL RADIUS

As derived by Braginskii, the equation that includes the time derivative of the channel radius ($\dot{a}$) is given as [12]:

$$2\pi^2\rho_o\xi a^3\dot{a}^3 = i^2/\sigma \quad (7)$$

Where (ρ_o) is the density of the undisturbed gas, and the value of (ξ) is taken as (4.5) for Air and Nitrogen [11, 12]. Equation (7) may be written in the form:

$$\frac{i^2}{\sigma} = \frac{\pi^2 \xi \rho_o}{4}(da^2/dt)^3 \qquad (8)$$

According to Braginskii, the electrical conductivity is approximately considered stationary [12], in this case, the integral form of Eq. (8) is [1, 11]:

$$a_{(t)}^2 = \left(\frac{4}{\pi^2 \rho_o \sigma \xi}\right)^{1/3} \int i^{2/3}\, dt \qquad (9)$$

Now, referring to Eq. (8) and considering the conductivity as a time-dependent quantity, we get the integral form of this equation as:

$$a_{(t)}^2 = \left(\frac{4}{\pi^2 \rho_o \xi}\right)^{1/3} \int \frac{i_{(t)}^{2/3}}{\sigma_{(t)}^{1/3}} dt \qquad (10)$$

Differentiating Eq. (10) with respect to (t), and using equations (2), (6), and (5), then rearranging and integrating again, we get:

$$a(t) = 0.4^{3/4}\left(\frac{12\,Pd}{\rho_o \xi A K_R \pi}\right)^{1/4} t^{5/4}$$

Or, $a(t) = f\, t^{5/4}$ (11)

Where $f = 0.4^{3/4}\left(\frac{12\,Pd}{\rho_o \xi A K_R \pi}\right)^{1/4}$.

Equation (11) determines the instantaneous channel radius with the conductivity regarded as a time- variable quantity (dynamic conductivity). The instantaneous conductivity; which is the conductivity at any instant during the resistive phase; can now be formulated by substituting equations (5) and (11) in Eq. (6), gives:

$$\sigma(t) = \frac{d}{\pi A f^2} t^{1/2} \qquad (12)$$

Where (f) as defined above.

Eq. (12) is used to determine the averaged conductivity ($\bar{\sigma}$)throughout the resistive phase period, which may be defined as:

$$\bar{\sigma} = \frac{\int_0^{\tau_R} \sigma(t) dt}{\int_0^{\tau_R} dt} \qquad (13)$$

Hence we get:

$$\bar{\sigma} = \frac{2\,d}{3\,\pi A f^2} \tau_R^{1/2} \qquad (14)$$

Now, replacing the steady conductivity (σ) in Eq. (9) with the averaged conductivity ($\bar{\sigma}$) given by Eq. (14), and using Eq. (2) to perform the integration we get:

$$a(t) = 0.375^{1/2}\left(\frac{d}{A}\right)^{1/3}\left(\frac{12\,P}{\pi^2 \rho_o \xi K_R \bar{\sigma}}\right)^{1/6} t^{4/3} \qquad (15)$$

Besides Eq. (11); which was derived based on a time-dependent conductivity; equation (15) provides an alternative way to determine the channel radius based on the averaged conductivity as a steady value.

It can be stated that the averaged conductivity provides a way to adopt the concept of steady conductivity adopted by Braginskii in determining the time evolution of the channel radius. In fact, what distinguishes the averaged conductivity from the steady conductivity proposed by Braginskii is that the former can be determined based on experimental parameters (which may differ from an experiment to another) while the latter was suggested as an approximate value [1, 11, 12, 13].Eq. (11) is used to determine the instantaneous induction of the channel through the resistive phase by making use of the relation [13]:

$$\frac{dL}{dt} = \frac{\mu_o}{2\pi}\frac{d}{a}\frac{da}{dt} \tag{16}$$

Where after performing the integration throughout the resistive phase period (τ_R), we get:

$$L(t) = 2.5\, d \times 10^{-7} \ln\left(\frac{\tau_R}{t}\right), (0< t \leq \tau_R) \tag{17}$$

The above expression of the channel inductance is somewhat different from that employed in earlier work [19]. The discharge inductance was also formulated as a function of the channel radius in another work [11] but this requires an initial guess value for the radius to perform the calculations.

4. Results and Discussion

Firstly, the present model was applied to illustrate the effect of gas pressure and gap width on the temporal evolution of both the channel radius and electrical conductivity. The value chosen for the circuit impedance is 29 Ω, and the gas in the gap is taken as Nitrogen [11].

Figure 1-a shows the temporal evolution of the channel radius throughout the resistive phase period for two values of gas pressure (P=1 and 3 atm.). The gap width is fixed at d= 0.1 mm. The values obtained from the breakdown voltage (V_B) and resistive phase period (τ_R) are shown in the caption of the figure. The corresponding curves of the electrical conductivity for this case are shown in Figure 1-b. In Figure 1-a, it is noticed that the radius increases at higher rates for the higher pressure condition (P= 3 atm.). This is in accordance with the higher breakdown potential (V_B) and the shorter resistive phase period (τ_R) associated with this condition, where this indicates a higher rate of energy (power) being fed to the discharge plasma. This is reflected in the form of a higher rise rate of the electrical conductivity associated with the high pressure condition as indicated in Figure 1-b. A similar comparison is shown in Figures 2-a and 2-b to illustrate the role of the gap width (d) in determining the temporal evolution of the radius and conductivity respectively. The gas pressure is held constant at P= 1 atm. It is realized that in the wider gap condition (d= 0.3 mm) we get slightly higher rates of radius growth (Figure 2-a), this can be explained on the basis of the relatively high breakdown voltage associated with the wide gap condition. On the other hand, the resistive phase period is relatively long for this condition, and this reduces the energy supply rate (power) delivered to the channel. Hence the role of the gap width in the channel radius evolution is less obvious compared with that of the gas pressure observed in Figure 1-a.

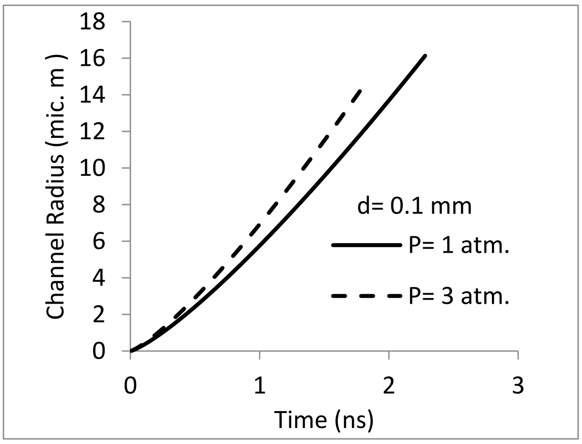

Figure 1-a: The effect of pressure on the time evolution of the channel radius at a fixed gap width. P= 1 atm.: V_B= 628 V, τ_R = 2.28 ns. P= 3 atm.: V_B = 1393 V, τ_R = 1.78 ns.

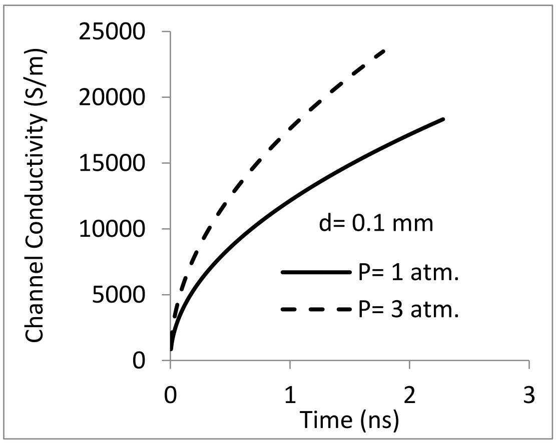

Figure 1-b: The effect of pressure on the time evolution of channel conductivity at a fixed gap width.

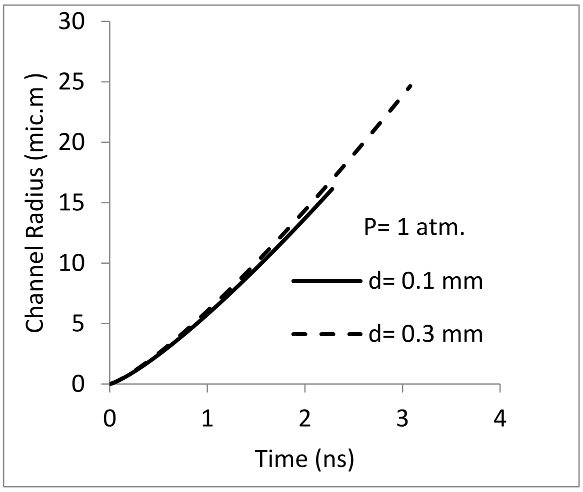

Figure 2-a: The effect of gap width on the time evolution of channel radius at a fixed pressure. d= 0.1 mm: V_B = 628 V, τ_R = 2.28 ns. d= 0.3 mm: V_B = 1393 V, τ_R = 3.08 ns.

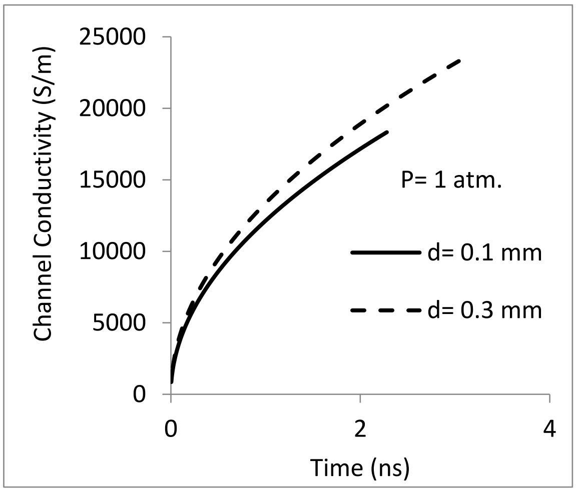

Figure 2-b: The effect of gap width on the time evolution of conductivity at a fixed pressure.

Figures 3-a and 3-b illustrate the dependence of the averaged conductivity on the gas pressure and gap width respectively. It can be noticed that the averaged conductivity; used in determining the instantaneous channel radius by Eq. (15); varies according to the experimental conditions. Hence, it is tending to believe that using the averaged conductivity as a steady value in determining the channel radius is more appropriate than merely assigning an approximated value [11, 12].

Figure 4 shows the channel radius evolution as determined by Eq. (11); where the conductivity is considered variable over time; and that determined by Eq. (15) where the averaged conductivity; determined by Eq. (14); is used as a steady conductivity value. The coincidence between the two curves indicates that the averaged conductivity is an appropriate counterpart of the time variable conductivity in determining the channel expansion. In Figure 5, the trends of two radius curves are shown, one is determined by the present model through Eq. (11); where the conductivity is considered variable; and the other is determined based on the empirical equation of Sorensen and Ristic for the channel resistance given by Eq. (4). The latter curve of the radius [a (t)] is obtained from Eq. (6) with Eq. (4) used for the resistance and Eq. (14) for the averaged conductivity. It can be noticed that the two curves match at early times, and their trends are similar (their correlation factor is 0.9978). Also, it is noticed that they are both concave curves, unlike those obtained from some other models which exhibit convex profiles [11, 20], and this should be expected since the Sorensen- Ristic exponential relation itself is employed in the present modeling.Regarding the channel inductance, Figures 6-a and 6-b illustrates the effect of gas pressure and gap width respectively. Since the inductance is strongly related to the rate of electric current variation, it is appropriate to implement similar comparisons for the electric current. The curves of the corresponding electric currents are shown in Figures 7-a and 7-b. It can be noticed that the inductance decreases throughout the resistive phase period and this is associated with an electric current rise. Taking a comprehensive look at Figures 6 and 7 one may deduce that increasing the pressure and decreasing the gap width act to improve the current rise rate pattern.

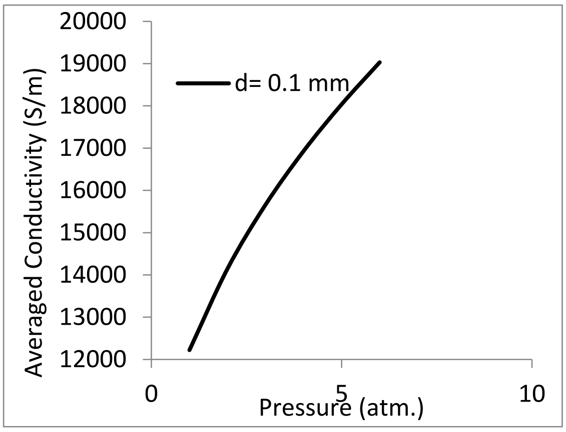

Figure 3-a: Averaged conductivity as a function of gas pressure. The gap width is held fixed.

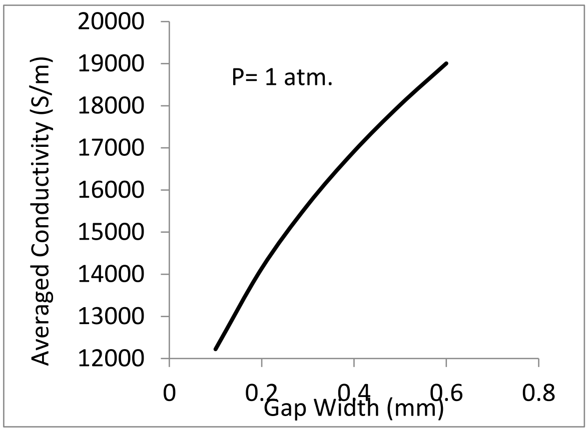

Figure 3-b: Averaged conductivity as a function of gap width. The gas pressure is held fixed.

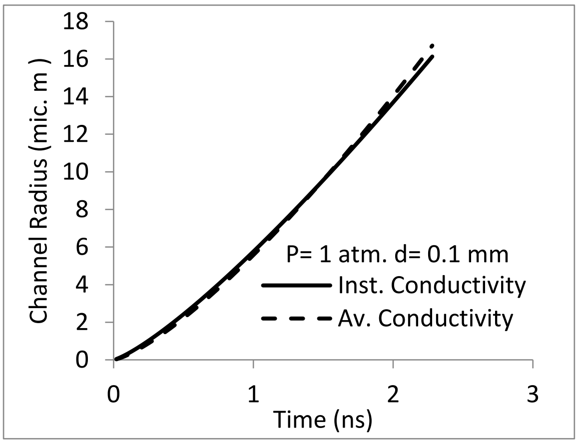

Figure 4: Channel radius versus time as determined using instantaneous and averaged conductivities. The value of the averaged conductivity for the case indicated is (1.2×10^4 S/m) as determined by eq. (14).

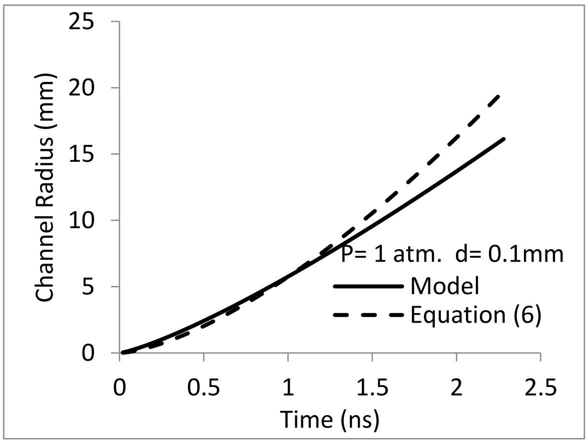

Figure 5: Channel radius as determined by the present model using a dynamic conductivity (solid curve), and that determined based on the empirical relation with an averaged conductivity (dashed curve).

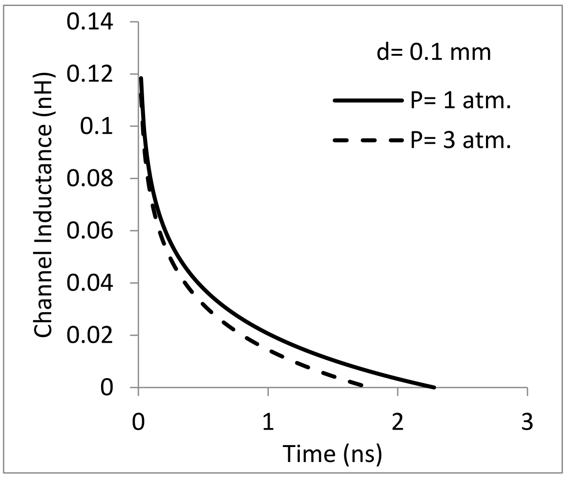

Figure 6-a: The effect of pressure on the channel induction variation through the resistive phase at a fixed gap width.

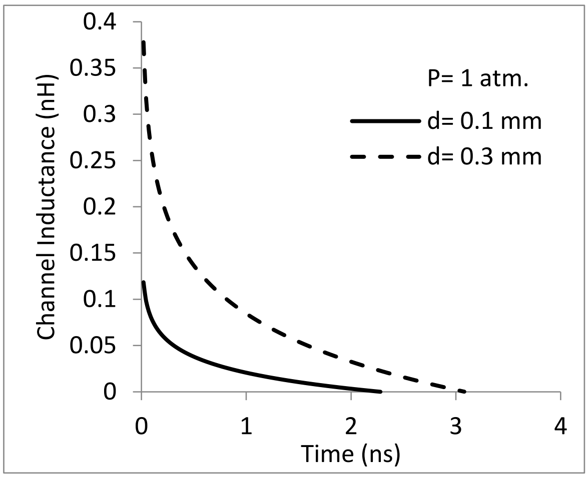

Figure 6-b: The effect of gap width on the channel induction variation through the resistive phase at a fixed pressure.

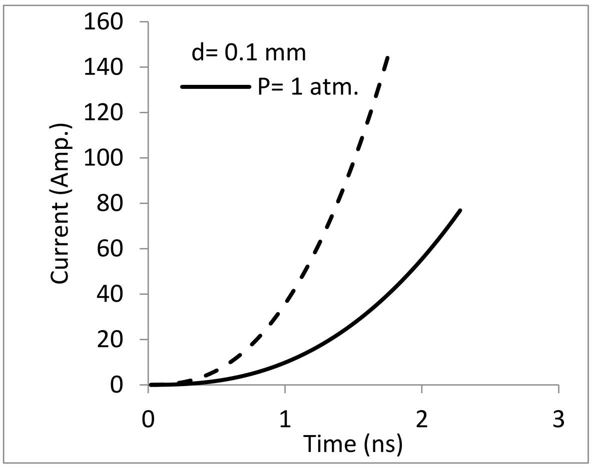

Figure. (7-a): The effect of pressure on the channel current variation through the resistive phase at a fixed gap width.

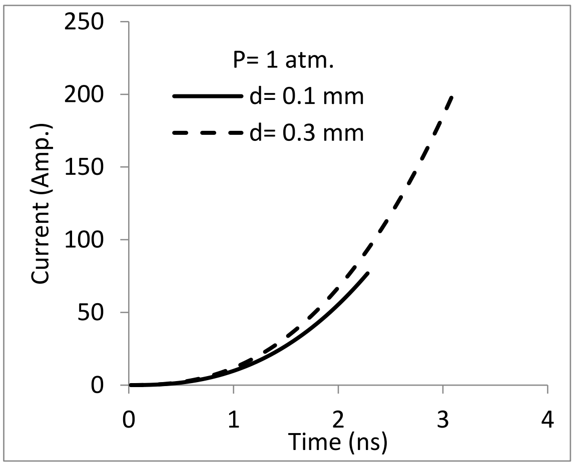

Figure. (7-b): The effect of gap width on the channel current variation through the resistive phase at a fixed pressure.

5. CONCLUSIONS

The present model needs to be compared with experimental results for quantitative verification. Unfortunately, the practical results in the range of nanosecond spark channels are very scarce to compare with. However, qualitatively, the temporal evolution curves for the channel radius, electrical conductivity, and inductance, obtained from the model have shown reasonable trends. The exponential behavior of the radius curves is in agreement with those obtained based on the empirical equation of Sorensen and Ristic. Regarding the electrical conductivity, the curves obtained by the present model show that in general, the conductivity increases during the breakdown. The trends of the electrical conductivity and inductance curves obtained from the model are similar to those obtained theoretically in early times in the discharge process since the time scale in that reference is in microseconds.

To summarize, the present model may be regarded as a simple tool to determine the temporal evolution of the channel radius, electrical conductivity, and inductance of the spark channel, as well as to examine the influence of both the gas pressure and gap width on the evolutionary patterns. The availability of experimental data regarding nanosecond and sub-nanosecond spark channel is necessary for further evaluation of this model as well as providing the required feedback for any possible developments.

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