Kinematics Analysis of 5 DOF Robotic Arm

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KEY WORDS
Robotic Arm, Forward Kinematics, Inverse Kinematics, Velocity Kinematics.

ABSTRACT
This paper presents the forward, inverse, and velocity kinematics analysis of a 5 DOF robotic arm. The Denavit-Hartenberg (DH) parameters are used to determination of the forward kinematics while an algebraic solution is used in the inverse kinematics solution to determine the position and orientation of the end effector. Jacobian matrix is used to calculate the velocity kinematics of the robotic arm. The movement of the robotic arm is accomplished using the microcontroller (Arduino Mega2560), which controlling on five servomotors of the robotic arm joints and one servo of the gripper. The position and orientation of the end effector are calculated using MATLAB software depending on the DH parameters. The results indicated the shoulder joint is more effect on the velocity of the robotic arm from the other joints, and the maximum error in the position of the end effector occurred with the z-axis and minimum error with the y-axis.

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1. Introduction
In recent years, the use of robotics in the manufacturing industries has increased tremendously. Robotics has enabled flexibility in manufacturing processes that enable the production of high-quality goods at relatively low cost [1]. The path planning of the robotic arm is depending on the kinematic analysis, involves both velocity kinematics and displacement kinematics. These kinematics analyses are a linear relationship between joint space and end-effector space and are essential for precise control and motion.
planning [2, 3]. Displacement Kinematics can be divided into two types: Forward kinematics and inverse kinematics [4], as shown in Figure 1. In this work, the forward, inverse, and velocity kinematics of the 5 DOF robotic arm was derived because of its importance in the movement of the arm in the pick and place process.

2. Related Works

AL-Tameemi and Hadi [5] presented the kinematics analysis (forward and inverse) for the 5 DOF robotic arm (Lab-Volt 5250). The D-H method is used to solve forward kinematics using the MATLAB software and analytical solution to solve the inverse kinematics.

Shabeeb and Mohammed [6] presented the forward kinematics analysis of the 5 DOF robotic arm (Lab-Volt 5150) and determined the position and orientation of the end effector using the D-H parameters. The MATLAB software was used to simulate the movement of the end effector.

Abaas and Abdulridha [7] presented the inverse kinematics solution for the robotic arm (Lab-Volt 5150) with 5 DOF, and the simulate of position and orientation of the end effector are accomplished by the MATLAB software. The results indicated the small error between the results extracted from RoboCIM software and the results from the MATLAB software.

Roshanianfard and Noguchi [8] investigated a new robotic arm with 5 DOF work to harvestings of heavy crops, and Solid works software was used to design and analysis the robotic arm. D-H method was used to solve the forward and inverse kinematics. The results showed the effectiveness of the investigated algorithm.

3. Kinematic Modelling

I. Forward Kinematics

The forward kinematics shows the transformation from one frame into another one, starting at the base and ending at the end-effector. A commonly used convention for selecting frames of reference in robotic applications is the Denavit-Hartenberg or DH convention, as shown in Figure 2. In this method, to set one reference frame relative to another, only four parameters are needed instead of six, which are normally required for 3D motion. These parameters are (di, ai, θi, and αi), which tell the location of a link-frame of the robot from a previous link-frame. The transformation matrix between two neighboring frames is expressed, as shown in Eq. (1) [9].

\[
T_i = R_{z,\theta_i} T_{z,\alpha_i} T_{x,a_i} R_{x,a_i}
\]

\[
T_i = \begin{bmatrix}
c\theta_i & -s\theta_i & 0 & 0 \\
s\theta_i & c\theta_i & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Where:

\[P_x: \text{Position of the end-effector in x-direction} = a_i \ c\theta_i\]

\[P_y: \text{Position of the end-effector in y-direction} = a_i \ s\theta_i\]

\[P_z: \text{Position of the end-effector in z-direction} = d_i\]

\[a_i: \text{Link length}\]

\[a_i: \text{Link twist}\]

\[d_i: \text{Link offset}\]

\[\theta_i: \text{Joint angle}\]

The parameters for the 5 DOF robotic arm used are listed in Table 1, where shows rotation about the z-axis, rotation about the x-axis, transition along the z-axis, and transition along the x-axis. By
substituting the D-H parameters in Table 1 into Eq (1), we can obtain the individual transformation matrices $T^0_1$ to $T^0_5$, and a global matrix of transformation $T^0_5$, as illustrated in Figure 3.

![Figure 1: Kinematic block diagram](image1)

![Figure 2: D-H frame](image2)

![Figure 3: Link coordinate diagram of the robotic arm](image3)

**Table 1: DH parameters for the robotic arm**

<table>
<thead>
<tr>
<th>Link</th>
<th>$a_i$ (mm)</th>
<th>$a_i$ (degree)</th>
<th>$d_i$ (mm)</th>
<th>$\theta_i$ (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>90</td>
<td>105</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>105</td>
<td>0</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>90</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>150</td>
<td>$\theta_5$</td>
</tr>
</tbody>
</table>

$$T^0_1 = R_x \theta_1 T_{z,d_1} T_{x,a_1} R_x \alpha_1$$

$$T^0_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$T^0_1 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c(90) & -s(90) & 0 \\ 0 & s(90) & c(90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
\[ T_1^0 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (2)

In a similar way will find the \( T_2^1, T_3^2, T_4^3, \) and \( T_5^4. \)

\[ T_2^1 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (3)

\[ T_3^2 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (4)

\[ T_4^3 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (5)

\[ T_5^4 = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (6)

The position and orientation for the wrist of the articulated robot can be obtained by multiply the matrices \( T_1^0, T_2^1, \) and \( T_3^2: \)

\[ T_3^0 = T_1^0 T_2^1 T_3^2 = \begin{bmatrix} c_1c_23 & -c_1s_23 & s_1 & c_1(a_3c_23 + a_2c_2) \\ s_1c_23 & -s_1s_23 & -c_1 & s_1(a_3c_23 + a_2c_2) \\ s_23 & c_23 & 0 & a_3s_23 + a_2s_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (7)

\[ T_5^0 = T_4^3 T_5^4 \\
= \begin{bmatrix} c_4c_5 & -s_5c_4 & s_4 & d_5s_4 \\ s_4c_5 & -s_4s_5 & -c_4 & -d_5c_4 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (8)

\[ T_5^0 = T_3^0 T_5^0 \\
= \begin{bmatrix} n_{11} & n_{12} & n_{13} & n_{14} \\ n_{21} & n_{22} & n_{23} & n_{24} \\ n_{31} & n_{32} & n_{33} & n_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \] (9)

\[ n_{11} = c_1c_23c_4 + s_1c_5 \]
\[ n_{12} = -s_5c_1234 + s_1c_5 \]
\[ n_{13} = c_1s_234 \]
\[ n_{14} = c_1(d_5s_234 + a_3c_23 + a_2c_2) \]
\[ n_{21} = s_1c_2345 - c_1s_5 \]
\[ n_{22} = -s_1c_2345 - c_1s_5 \]
\[ n_{23} = s_1c_3234 \]
\[ n_{24} = s_1(d_5s_234 + a_3c_23 + a_2c_2) \]
\[ n_{31} = c_5s_234 \]
\[ n_{32} = -s_5s_234 \]
\[ n_{33} = -c_234 \]
\[ n_{34} = -d_5c_234 + a_3s_23 + a_2s_2 + d_1 \]
II. Inverse kinematics

A geometric approach or algebraic method can obtain the inverse kinematics of the robot arm. In this work, an algebraic method is used to obtain the inverse kinematics of a 5 DOF robotic arm. To find the first joint will be using the Eq. (9):

\[
[T_1^0]^{-1} T_5^0 = [T_1^0]^{-1} T_1^0 T_2^0 T_3^0 T_4^0 T_5^0
\]

The equation above become:

\[
\begin{bmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\
\alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\
\alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
c_1 n_{11} + s_1 n_{21} & -s_1 n_{12} + c_1 n_{22} \\
0 & 0 \\
c_1 n_{13} + s_1 n_{23} & -s_1 n_{14} + c_1 n_{24} \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
c_1 x + s_1 y \\
-c_1 x + c_1 y \\
z - d_1
\end{bmatrix}
\]

(10)

\[
\begin{bmatrix}
c_1 x + s_1 y \\
-c_1 x + c_1 y \\
z - d_1
\end{bmatrix}
\]

(11)

From the equations (11, 12) we obtained the \( \theta_1 \)

\[\therefore \theta_1 = \text{atan}(y, x) \]

(13)

In a similar way will find the other joint angles:

\[
[T_1^0 T_2^0]^{-1} T_5^0 = T_2^0 T_3^0 T_4^0 T_5^0
\]

\[
[T_1^0 T_2^0 T_3^0]^{-1} T_5^0 = T_3^0 T_4^0 T_5^0
\]

\[
[T_1^0 T_2^0 T_3^0 T_4^0]^{-1} T_5^0 = T_4^0 T_5^0
\]

To find the \( \theta_2 \)

\[
c_1 x + s_1 y = -s_234 d_5 + c_23 a_4 + c_2 a_3
\]

\[
c_1 x + s_1 y = -s_234 d_5 + c_23 a_4 + c_2 a_3
\]

\[
c_1 x + s_1 y = -s_234 d_5 + c_23 a_4 + c_2 a_3
\]

\[
c_2 = \frac{(c_3 a_4 + a_3)}{(c_3 a_4 + a_3)}
\]

\[
z - d_1 = c_234 d_5 + s_23 a_4 + s_2 a_3
\]

\[
z - d_1 = c_234 d_5 + s_23 a_4 + s_2 a_3
\]

\[
z - d_1 = c_234 d_5 + s_23 a_4 + s_2 a_3
\]
III. Velocity kinematics

Jacobian matrix is used to calculate the velocity kinematics using a matrix, which depends on the changes of the joints velocities into Cartesian velocities. This matrix is important in control of the movement of the robotic arm, used to achieve smooth path planning, and used to determine the dynamic equation. The relationships between the joint velocity and the linear and angular velocity of the end effector are shown following:

\[ \dot{p} = J_p(\theta)\dot{\theta} \]
\[ \dot{w} = J_w(\theta)\dot{\theta} \]

By combining, the equations (19, 20) will give the Jacobian matrix:

\[ J_\theta = \begin{bmatrix} J_{p1} & \cdots & J_{pn} \\ J_{w1} & \cdots & J_{wn} \end{bmatrix} \]

Where:
\[ \theta \] : Joint angle.
\[ \dot{\theta} \] : joint velocity.
\[ \dot{p} \] : linear velocity of the end effector.
\[ \dot{w} \] : angular velocity of the end effector.

The number of rows in the Jacobian matrix equal to the number of DOF in the cartesian coordinate (three linear and three angular) while the number of columns equal to the number of DOF in the joint.

The matrix of Jacobian can be obtained using the following equations:

\[ J_{pi} = \begin{cases} z_{i-1} \times (o_n - a_{i-1}) & \text{for revolute joint } (i) \\ z_{i-1} & \text{for prismatic joint } (i) \end{cases} \]

\[ J_{wi} = \begin{cases} z_{i-1} & \text{for revolute joint } (i) \\ 0 & \text{for prismatic joint } (i) \end{cases} \]

Where:
\[ J_{pi} \] : Linear Jacobian matrix.
\[ J_{wi} \] : Angular Jacobian matrix.

For the 5 DOF robotic arm used, the Jacobian matrix will be in the form:

\[ J(\theta) = \begin{bmatrix} z_0 \times (o_5 - o_0) & z_1 \times (o_5 - o_1) & \cdots & z_2 \times (o_5 - o_2) \\ z_0 & z_1 & \cdots & z_2 \\ z_3 \times (o_5 - o_3) & z_4 \times (o_5 - o_4) \end{bmatrix} \]
From the forward kinematics can be calculated the linear part of \( f(\theta) \), the value of \((a_n - a_{n-1})\) for 5 DOF robotic arm used is:

\[
\begin{bmatrix}
a_0 \\
a_1 \\
a_2 \\
a_3 \\
a_4 \\
a_5 \\
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
1 \\
1 \\
\end{bmatrix}
\]

Also, the angular part of \( J(\theta) \) can be obtained using the forward kinematics, from the equations (2-9) the values of \( z_i \) are:

\[
\begin{bmatrix}
z_0 \\
z_1 \\
z_2 \\
z_3 \\
z_4 \\
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 \\
0 & 0 \\
\end{bmatrix}
\]

Then

\[
\begin{aligned}
z_0 \times (a_5 - a_0) &= \begin{bmatrix}
-d_5 s_1 s_{234} - a_3 s_1 c_{23} - a_2 s_1 c_2 \\
d_5 c_1 s_{234} + a_3 c_1 c_{23} + a_2 c_1 c_2 \\
0 \\
\end{bmatrix} = f_{11} \\
z_1 \times (a_5 - a_1) &= \begin{bmatrix}
d_5 c_1 c_{234} - a_3 c_1 s_{23} - a_2 s_2 c_1 \\
d_5 s_1 c_{234} - a_3 s_1 s_{23} - a_2 s_2 s_1 \\
d_5 s_{234} + a_3 c_{23} + a_2 c_2 \\
\end{bmatrix} = f_{12} \\
z_2 \times (a_5 - a_2) &= \begin{bmatrix}
d_5 c_1 c_{234} - a_3 c_1 s_{23} \\
d_5 s_1 c_{234} - a_3 s_1 s_{23} \\
d_5 s_{234} + a_3 c_{23} \\
\end{bmatrix} = f_{13} \\
z_3 \times (a_5 - a_3) &= \begin{bmatrix}
d_5 c_1 c_{234} \\
d_5 s_1 c_{234} \\
d_5 s_{234} \\
\end{bmatrix} = f_{14} \\
z_4 \times (a_5 - a_4) &= \begin{bmatrix}
0 \\
0 \\
0 \\
\end{bmatrix} = f_{15}
\end{aligned}
\]

The Jacobian matrix will become:
\[ J(\theta) = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ 0 & s_1 & s_1 & c_1 & s_2s_3 \\ 0 & -c_1 & -c_1 & -c_1 & s_2s_3 \\ 1 & 0 & 0 & 0 & -c_2s_3 \end{bmatrix} \] (31)

4. Experimental Work

The robotic arm system is designed and implemented, as shown in Figure 4, which contains robotic arm (5 DOF), Servomotors, power supply, regulators, and microcontroller (Arduino). The forward and inverse kinematics were used to obtain the required angles of joints that work to move the robotic arm and perform the picking of the object from the conveyor belt to a specified position. In addition, the Jacobian method was used to calculate the velocity of the end-effector.

5. Results and Discussion

The Kinematics Analysis of the 5 DOF robotic arm shown in Figure 4 is applied to perform several cases of movement, and the results of the position and orientation of the end effector of the robotic arm were listed in Table 2, and Figure 5 shows the process. The velocities of these cases were calculated using Eq. (22). Furthermore, these velocities were plotted with time, as shown in Figure 6. From the results, indicating the derived kinematics analysis of 5 DOF robotic arm was correct to perform the movement of the robotic arm with little small error.

![Figure 4: The robotic arm system](image)

<table>
<thead>
<tr>
<th>Cases No.</th>
<th>Joint Angles (degree)</th>
<th>End Effector Position (True) (mm)</th>
<th>End Effector Position (Theoretically) (mm)</th>
<th>Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[\begin{align*} \theta_1 &amp;= 86 \ \theta_2 &amp;= 130 \ \theta_3 &amp;= 125 \ \theta_4 &amp;= 60 \ \theta_5 &amp;= 90 \end{align*}]</td>
<td>[\begin{align*} P_x &amp;= -21 \ P_y &amp;= 308 \ P_z &amp;= 172 \end{align*}]</td>
<td>[\begin{align*} P_x &amp;= -21.05 \ P_y &amp;= 308.3 \ P_z &amp;= 172.5 \end{align*}]</td>
<td>[\frac{</td>
</tr>
<tr>
<td>2</td>
<td>[\begin{align*} \theta_1 &amp;= 144 \ \theta_2 &amp;= 110 \ \theta_3 &amp;= 135 \ \theta_4 &amp;= 54 \ \theta_5 &amp;= 90 \end{align*}]</td>
<td>[\begin{align*} P_x &amp;= 220 \ P_y &amp;= 161 \ P_z &amp;= 220 \end{align*}]</td>
<td>[\begin{align*} P_x &amp;= 221.4 \ P_y &amp;= 160.9 \ P_z &amp;= 218 \end{align*}]</td>
<td>[\frac{</td>
</tr>
<tr>
<td>3</td>
<td>[\begin{align*} \theta_1 &amp;= 15 \ \theta_2 &amp;= 90 \ \theta_3 &amp;= 155 \ \theta_4 &amp;= 53 \ \theta_5 &amp;= 90 \end{align*}]</td>
<td>[\begin{align*} P_x &amp;= -230 \ P_y &amp;= 61 \ P_z &amp;= 220 \end{align*}]</td>
<td>[\begin{align*} P_x &amp;= -229.2 \ P_y &amp;= 61.43 \ P_z &amp;= 221.7 \end{align*}]</td>
<td>[\frac{</td>
</tr>
<tr>
<td>4</td>
<td>[\begin{align*} \theta_1 &amp;= 153 \ \theta_2 &amp;= 96 \end{align*}]</td>
<td>[\begin{align*} P_x &amp;= 220 \end{align*}]</td>
<td>[\begin{align*} P_x &amp;= 220.5 \ P_y &amp;= 220.5 \end{align*}]</td>
<td>[\frac{</td>
</tr>
</tbody>
</table>
\begin{array}{c|c|c}
\theta_1 &=& 148 \\
\theta_2 &=& 52 \\
\theta_3 &=& 90 \\
\theta_4 &=& \frac{111}{112} \\
P_x &=& 220 \\
P_y &=& 222.5 \\
P_z &=& 1.136 \\
\end{array}

Figure 5: The movement of cases
Figure 6: End-effector velocity of 5 DOF robotic arm at $\dot{\theta} = 20^\circ$/sec

From Figure 6 of velocity with time, it can be observed the maximum velocity was $(8.32 \times 10^3)$ mm/sec for joint 2; this means the shoulder joint is essential in controlling the movement of the robotic arm. Table 2 showed the maximum value of error in the position of the end-effector was $(1.136 \%)$ with the z-axis in case (4), that is because of the effect of the weight of the servomotors and arm of the robotic arm. Moreover, the minimum error of the end-effector position was $(0.06 \%)$ with the y-axis in case (2).

6. Conclusions

From the extracted results can conclude of the study as follows:
The derivative equations of the forward and inverse kinematics gave good results in determining the position and orientation of the robotic arm.
The maximum errors in position were occurred in the z-axis due to the weight of both the servomotors and links of the robotic arm, and the minimum errors occurred in the y-axis.
The work leads to perform the pick and place process accurately.
References


